

Enriques Moonshine

TOHRU EGUCHI

*Department of Physics and Research Center for Mathematical Physics,
Rikkyo University, Tokyo 171-8501, Japan.*

KAZUHIRO HIKAMI

Faculty of Mathematics, Kyushu University, Fukuoka 819-0395, Japan.

ABSTRACT. We propose a new moonshine phenomenon associated with the elliptic genus of the Enriques surface ($\frac{1}{2}$ of the elliptic genus of $K3$) with the symmetry group given by the Mathieu group M_{12} .

1. Mathieu moonshine

Recently a new moonshine phenomenon associated with the elliptic genus of $K3$ surface has been discovered and is receiving some attentions. It was first observed in [7] that when one expands the elliptic genus of $K3$ in terms of irreducible characters of $\mathcal{N} = 4$ superconformal algebra (SCA) the expansion coefficients $A(n)$ at lower values of n are decomposed into a sum of dimensions of irreducible representations (irreps.) of the Mathieu group M_{24} . Subsequently the twisted elliptic genera of $K3$ surface for each conjugacy class g of M_{24} (analogues of McKay–Thompson series of monstrous moonshine) have been constructed and used to determine systematically the decomposition of expansion coefficients up to very high values of n (~ 1000) [1, 5, 8, 9]. Finally a mathematical proof has been given to show that expansion coefficients are in fact decomposed into a sum of dimensions of irreps. of M_{24} with positive and integral multiplicities for all values of n [10]. Thus the “Mathieu moonshine” phenomenon has now been established although its physical or mathematical origin is not yet explained.

We present the character table and list of conjugacy classes of M_{24} in Tables 1 and 2. We also present the data of the decomposition of expansion coefficients $A(n)$ of elliptic genus of $K3$

$$Z_{K3}(z; \tau) = 24 \operatorname{ch}_{h=\frac{1}{4}, \ell=0}^{\tilde{R}}(z; \tau) + \sum_{n=0}^{\infty} A(n) \operatorname{ch}_{h=n+\frac{1}{4}, \ell=\frac{1}{2}}^{\tilde{R}}(z; \tau) \quad (1.1)$$

E-mail addresses: tohru.eguchi@gmail.com, khikami@gmail.com.

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into irreps. of M_{24} in Table 3. Note that here Z_{K3} denotes the elliptic genus of $K3$ and $\text{ch}_{h=\frac{1}{4},\ell}^{\tilde{R}}$ and $\text{ch}_{h=n+\frac{1}{4},\ell}^{\tilde{R}}$ are massless (BPS) and massive (non-BPS) characters (with $h = n + \frac{1}{4}$ and spin- ℓ) of $\mathcal{N} = 4$ SCA in R-sector with $(-1)^F$ insertion. For later use we also record the data of expansion coefficients $A_g(n)$ of twisted elliptic genera $Z_g(z; \tau)$ of $K3$ for each conjugacy class $g \in M_{24}$

$$Z_g(z; \tau) = \chi_g \text{ch}_{h=\frac{1}{4},\ell=0}^{\tilde{R}}(z; \tau) + \sum_{n=0}^{\infty} A_g(n) \text{ch}_{h=n+\frac{1}{4},\ell=\frac{1}{2}}^{\tilde{R}}(z; \tau), \quad (1.2)$$

in Table 4. Note that $A(n) \equiv A_{1A}(n)$.

Recently there has been an attempt at generalizing Mathieu moonshine [2] based on suitable Jacobi forms with higher values of indices > 1 and again expanding them in terms of $\mathcal{N} = 4$ superconformal characters using the data of [4]. This “umbral moonshine” sequence has smaller symmetry groups than M_{24} . Unfortunately, its Jacobi forms do not correspond to elliptic genera of any complex manifolds and the connection to geometry is not clear in umbral moonshine. In [6] we have discussed a still another example of moonshine based on $\mathcal{N} = 2$ SCA instead of $\mathcal{N} = 4$.

2. Enriques moonshine

In this paper we want to propose a new example of moonshine phenomenon which may be called as “Enriques moonshine”. It is defined by the elliptic genus of Enriques surface expanded in terms of $\mathcal{N} = 4$ characters. Its symmetry group is M_{12} . Recall that Enriques surface is closely related to $K3$: it is obtained by quotienting $K3$ by a fix-point free involution and has an Euler number 12. Its elliptic genus is one half of that of $K3$

$$Z_{\text{Enriques}}(z; \tau) = \frac{1}{2} Z_{K3}(z; \tau) = 4 \left[\left(\frac{\theta_{10}(z; \tau)}{\theta_{10}(0; \tau)} \right)^2 + \left(\frac{\theta_{00}(z; \tau)}{\theta_{00}(0; \tau)} \right)^2 + \left(\frac{\theta_{01}(z; \tau)}{\theta_{01}(0; \tau)} \right)^2 \right]. \quad (2.1)$$

Enriques moonshine is motivated by the following simple considerations.

1. It is known that in the case of Mathieu moonshine the expansion coefficients $A(n)$ are always even for any $n \geq 1$: this is because (i) when the decomposition of $A(n)$ contains a complex representation of M_{24} , it also contains its complex conjugate representation, and (ii) when $A(n)$ contains a real representation its multiplicity is always even [10].
2. Thus in order to keep integrality of the decomposition when we divide by 2 the $K3$ elliptic genus we just need to find a subgroup G of M_{24} where all the complex representations of M_{24} become real representations of G . It turns out that this is the case of M_{12} .
3. Geometrical considerations on Enriques surface suggests the relevance of the symmetry group M_{12} [11].

Let us first derive the decomposition of M_{24} representations (reps.) into those of M_{12} in order to examine the reality of representations. For this purpose we want to make a correspondence between the conjugacy classes of the two groups. In Table 5 we

list the conjugacy classes of M_{12} and their permutation representations. We recall that Mathieu group M_{24} is the symmetry group of Golay code and permutes dodecads into each other. M_{12} is the subgroup of M_{24} which fixes a dodecad [3]. Conjugacy class of 2A of M_{12} , for instance, has a cycle shape 2^6 and it is natural that this corresponds to the conjugacy class 2B of M_{24} with a cycle shape 2^{12} . Thus in general a class g of M_{12} should correspond to a class g' of M_{24} whose cycle shape is the square of that of g . There are exceptions to this rule when there exists a non-trivial outer automorphism between conjugacy classes of M_{12} . From the Table 5 we note that the sizes of conjugacy classes are equal for the pair 4A, 4B and 8A, 8B and 11A, 11B. It is known [3] that these pairs are tied by a non-trivial outer automorphism σ . If one takes a class g of M_{12} the corresponding class of M_{24} should become $g \cup \sigma(g)$. In the case of $g = 4A$, $\sigma(4A) = 4B$, for instance, the cycle shape of $g \cup \sigma(g)$ equals $4^2 2^2 \cup 4^2 1^4$ and that of g' becomes $4^4 2^2 1^4$ which is class 4B of M_{24} . Thus 4A, 4B of M_{12} both should correspond to 4B of M_{24} . In this way we can construct the following table of correspondences.

| | | | | | | | | | | | | | | | |
|-----------------|----|----|----|----|----|----|----|----|----|----|----|----|-----|-----|-----|
| $g \in M_{12}$ | 1A | 2A | 2B | 3A | 3B | 4A | 4B | 5A | 6A | 6B | 8A | 8B | 10A | 11A | 11B |
| $g' \in M_{24}$ | 1A | 2B | 2A | 3A | 3B | 4B | 4B | 5A | 6B | 6A | 8A | 8A | 10A | 11A | 11A |

(2.2)

Let us now determine the branching rule of the irreps. of M_{24} into those of M_{12} . We consider the following “inner product” of character tables of M_{24} and M_{12} to derive the multiplicity of a representation r of M_{12} contained in the representation R of M_{24}

$$\sum_g (\chi_{M_{24}})_R^{t(g)} (\chi_{M_{12}})_g^{-1} = \text{multiplicity of irrep. } r \text{ in irrep. } R \quad (2.3)$$

Here $t(g) = g'$ of (2.2), and $(\chi_{M_{12}})^{-1}$ is the inverse of the character table of M_{12} in the sense of a matrix. Using the character tables of M_{24} , M_{12} in Tables 1, 6 we find the above multiplicities given by Table 7. Note that as we mentioned already, decomposition of complex representations of M_{24} contain only real representations of M_{12} or the sum of complex and complex conjugate representations of M_{12} .

Therefore if we substitute M_{24} reps. by their M_{12} decompositions in the Mathieu moonshine of Table 3, and divide by an overall factor 2, we maintain the integrality of multiplicities of M_{12} representations. One obtains the decomposition of the elliptic genus of Enriques surface given in terms of M_{12} reps.. See Table 8.

There is in fact a more elegant way to derive the decomposition of Enriques elliptic genus. This is to use the method of twisted genus. We have at hand the twisted genera for all conjugacy classes in Mathieu moonshine (tabulated in [5]) and we can use these results. We assume twisted elliptic genera for Enriques moonshine are one half of those of Mathieu moonshine of the corresponding conjugacy classes

$$Z_g^{\text{Enriques}}(z; \tau) = \frac{1}{2} Z_{t(g)}^{K3}(z; \tau) \quad \text{for all conjugacy classes } g \text{ of } M_{12} \quad (2.4)$$

Then by introducing the expansion coefficients $A_g^{\text{Enriques}}(n)$ for all classes $g \in M_{12}$

$$Z_g^{\text{Enriques}}(z; \tau) = \chi_g^{\text{Enriques}} \text{ch}_{h=\frac{1}{4}, \ell=0}(z; \tau) + \sum_{n=0}^{\infty} A_g^{\text{Enriques}}(n) \text{ch}_{h=n+\frac{1}{4}, \ell=\frac{1}{2}}(z; \tau) \quad (2.5)$$

where χ_g^{Enriques} is the Euler number $\chi_g^{\text{Enriques}} = Z_g^{\text{Enriques}}(0; \tau)$,

| $g \in M_{12}$ | 1A | 2A | 2B | 3A | 3B | 4A | 4B | 5A | 6A | 6B | 8A | 8B | 10A | 11A | 11B |
|----------------------------|----|----|----|----|----|----|----|----|----|----|----|----|-----|-----|-----|
| χ_g^{Enriques} | 12 | 0 | 4 | 3 | 0 | 2 | 2 | 2 | 0 | 1 | 1 | 1 | 0 | 1 | 1 |

we obtain the multiplicity for the M_{12} representaion r at level n

$$\sum_g \frac{1}{|G|} n_g \overline{\chi_{M_{12}r}}^g A_g^{\text{Enriques}}(n) = c_r^{\text{Enriques}}(n). \quad (2.6)$$

Here $|G|$ denotes the order of $M_{12}(=95040)$ and n_g is the size of class g . The above formula exactly reproduce the data of Table 8.

3. Discussions

In this article we have taken the one half of Mathieu moonshine and obtained Enriques moonshine. Relevance of Enriques surface in string theory is a delicate issue since its canonical class does not quite vanish while it carries a Ricci flat Kähler metric. We disregard such questions in this paper and are concerned with the possibility of the action of some symmetry groups on its elliptic genus. It will be interesting to see if it is possible to take one half of the umbral moonshine series.

Note added: After this paper has been submitted to arXiv we came to know the paper [12] by S. Govindarajan where the group M_{12} is used as the symmetry group of Mathieu moonshine. In this paper the relation (2.2) between the conjugacy classes of M_{24} and M_{12} has been obtained. Also the multiplicities of irreps. of M_{12} in the decomposition of expansion coefficients $A(n)$ at smaller values of n have been obtained in agreement with our results of Enriques moonshine upto an overall factor 2. We thank S. Govindarajan for informing us of this paper.

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TABLE 1. character table of M_{24} . $|M_{24}| = 244823040$

| | 1A | 2A | 2B | 3A | 3B | 4A | 4B | 4C | 5A | 6A | 6B | 7A | 7B | 8A | 10A | 11A | 12A | 12B | 14A | 14B | 15A | 15B | 21A | 21B | 23A | 23B |
|-------------|-------|-----|-----|-----|----|----|----|----|----|----|----|--------------------------|--------------------------|----|-----|-----|-----|-----|--------------------------|--------------------------|---------------------------|---------------------------|--------------------------|--------------------------|---------------------------|---------------------------|
| χ_1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| χ_2 | 23 | 7 | -1 | 5 | -1 | -1 | 3 | -1 | 3 | 1 | -1 | 2 | 2 | 1 | -1 | 1 | -1 | -1 | 0 | 0 | 0 | 0 | -1 | -1 | 0 | 0 |
| χ_3 | 45 | -3 | 5 | 0 | 3 | -3 | 1 | 1 | 0 | 0 | -1 | $\frac{-1-i\sqrt{7}}{2}$ | $\frac{-1-i\sqrt{7}}{2}$ | -1 | 0 | 1 | 0 | 1 | $\frac{-1+i\sqrt{7}}{2}$ | $\frac{-1-i\sqrt{7}}{2}$ | 0 | 0 | $\frac{-1+i\sqrt{7}}{2}$ | $\frac{-1-i\sqrt{7}}{2}$ | -1 | -1 |
| χ_4 | 45 | -3 | 5 | 0 | 3 | -3 | 1 | 1 | 0 | 0 | -1 | $\frac{-1-i\sqrt{7}}{2}$ | $\frac{-1+i\sqrt{7}}{2}$ | -1 | 0 | 1 | 0 | 1 | $\frac{-1-i\sqrt{7}}{2}$ | $\frac{-1+i\sqrt{7}}{2}$ | 0 | 0 | $\frac{-1-i\sqrt{7}}{2}$ | $\frac{-1+i\sqrt{7}}{2}$ | -1 | -1 |
| χ_5 | 231 | 7 | -9 | -3 | 0 | -1 | -1 | 3 | 1 | 1 | 0 | 0 | 0 | -1 | 1 | 0 | -1 | 0 | 0 | 0 | $\frac{-1+i\sqrt{15}}{2}$ | $\frac{-1-i\sqrt{15}}{2}$ | 0 | 0 | 1 | 1 |
| χ_6 | 231 | 7 | -9 | -3 | 0 | -1 | -1 | 3 | 1 | 1 | 0 | 0 | 0 | -1 | 1 | 0 | -1 | 0 | 0 | 0 | $\frac{-1-i\sqrt{15}}{2}$ | $\frac{-1+i\sqrt{15}}{2}$ | 0 | 0 | 1 | 1 |
| χ_7 | 252 | 28 | 12 | 9 | 0 | 4 | 4 | 0 | 2 | 1 | 0 | 0 | 0 | 0 | 2 | -1 | 1 | 0 | 0 | 0 | -1 | -1 | 0 | 0 | -1 | -1 |
| χ_8 | 253 | 13 | -11 | 10 | 1 | -3 | 1 | 1 | 3 | -2 | 1 | 1 | 1 | -1 | -1 | 0 | 0 | 1 | -1 | -1 | 0 | 0 | 1 | 1 | 0 | 0 |
| χ_9 | 483 | 35 | 3 | 6 | 0 | 3 | 3 | 3 | -2 | 2 | 0 | 0 | 0 | -1 | -2 | -1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| χ_{10} | 770 | -14 | 10 | 5 | -7 | 2 | -2 | -2 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | -1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | $\frac{-1+i\sqrt{23}}{2}$ | $\frac{-1-i\sqrt{23}}{2}$ |
| χ_{11} | 770 | -14 | 10 | 5 | -7 | 2 | -2 | -2 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | -1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | $\frac{-1-i\sqrt{23}}{2}$ | $\frac{-1+i\sqrt{23}}{2}$ |
| χ_{12} | 990 | -18 | -10 | 0 | 3 | 6 | 2 | -2 | 0 | 0 | -1 | $\frac{-1+i\sqrt{7}}{2}$ | $\frac{-1-i\sqrt{7}}{2}$ | 0 | 0 | 0 | 0 | 1 | $\frac{-1+i\sqrt{7}}{2}$ | $\frac{-1-i\sqrt{7}}{2}$ | 0 | 0 | $\frac{-1+i\sqrt{7}}{2}$ | $\frac{-1-i\sqrt{7}}{2}$ | 1 | 1 |
| χ_{13} | 990 | -18 | -10 | 0 | 3 | 6 | 2 | -2 | 0 | 0 | -1 | $\frac{-1-i\sqrt{7}}{2}$ | $\frac{-1+i\sqrt{7}}{2}$ | 0 | 0 | 0 | 0 | 1 | $\frac{-1-i\sqrt{7}}{2}$ | $\frac{-1+i\sqrt{7}}{2}$ | 0 | 0 | $\frac{-1-i\sqrt{7}}{2}$ | $\frac{-1+i\sqrt{7}}{2}$ | 1 | 1 |
| χ_{14} | 1035 | 27 | 35 | 0 | 6 | 3 | -1 | 3 | 0 | 0 | 2 | -1 | -1 | 1 | 0 | 1 | 0 | 0 | -1 | -1 | 0 | 0 | -1 | -1 | 0 | 0 |
| χ_{15} | 1035 | -21 | -5 | 0 | -3 | 3 | 3 | -1 | 0 | 0 | 1 | $-1+i\sqrt{7}$ | $-1-i\sqrt{7}$ | -1 | 0 | 1 | 0 | -1 | 0 | 0 | 0 | 0 | $\frac{-1+i\sqrt{7}}{2}$ | $\frac{-1-i\sqrt{7}}{2}$ | 0 | 0 |
| χ_{16} | 1035 | -21 | -5 | 0 | -3 | 3 | 3 | -1 | 0 | 0 | 1 | $-1-i\sqrt{7}$ | $-1+i\sqrt{7}$ | -1 | 0 | 1 | 0 | -1 | 0 | 0 | 0 | 0 | $\frac{-1-i\sqrt{7}}{2}$ | $\frac{-1+i\sqrt{7}}{2}$ | 0 | 0 |
| χ_{17} | 1265 | 49 | -15 | 5 | 8 | -7 | 1 | -3 | 0 | 1 | 0 | -2 | -2 | 1 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| χ_{18} | 1771 | -21 | 11 | 16 | 7 | 3 | -5 | -1 | 1 | 0 | -1 | 0 | 0 | -1 | 1 | 0 | 0 | -1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| χ_{19} | 2024 | 8 | 24 | -1 | 8 | 8 | 0 | 0 | -1 | -1 | 0 | 1 | 1 | 0 | -1 | 0 | -1 | 0 | 1 | 1 | -1 | -1 | 1 | 1 | 0 | 0 |
| χ_{20} | 2277 | 21 | -19 | 0 | 6 | -3 | 1 | -3 | -3 | 0 | 2 | 2 | 2 | -1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -1 | 0 | 0 |
| χ_{21} | 3312 | 48 | 16 | 0 | -6 | 0 | 0 | 0 | -3 | 0 | -2 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | -1 | -1 | 0 | 0 | 1 | 1 | 0 | 0 |
| χ_{22} | 3520 | 64 | 0 | 10 | -8 | 0 | 0 | 0 | 0 | -2 | 0 | -1 | -1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | -1 | -1 | 1 | 1 |
| χ_{23} | 5313 | 49 | 9 | -15 | 0 | 1 | -3 | -3 | 3 | 1 | 0 | 0 | 0 | -1 | -1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| χ_{24} | 5544 | -56 | 24 | 9 | 0 | -8 | 0 | 0 | -1 | 1 | 0 | 0 | 0 | 0 | -1 | 0 | 1 | 0 | 0 | 0 | -1 | -1 | 0 | 0 | 1 | 1 |
| χ_{25} | 5796 | -28 | 36 | -9 | 0 | -4 | 4 | 0 | 1 | -1 | 0 | 0 | 0 | 0 | 1 | -1 | -1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| χ_{26} | 10395 | -21 | -45 | 0 | 0 | 3 | -1 | 3 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -1 |

| g | size | cycle shape |
|-----|----------|--------------------|
| 1A | 1 | 1^{24} |
| 2A | 11385 | $1^8 2^8$ |
| 2B | 31878 | 2^{12} |
| 3A | 226688 | $1^6 3^6$ |
| 3B | 485760 | 3^8 |
| 4A | 637560 | $2^4 4^4$ |
| 4B | 1912680 | $1^4 2^2 4^4$ |
| 4C | 2550240 | 4^6 |
| 5A | 4080384 | $1^4 5^4$ |
| 6A | 10200960 | $1^2 2^2 3^2 6^2$ |
| 6B | 10200960 | 6^4 |
| 7A | 5829120 | $1^3 7^3$ |
| 7B | 5829120 | $1^3 7^3$ |
| 8A | 15301440 | $1^2 2^1 4^1 8^2$ |
| 10A | 12241152 | $2^2 10^2$ |
| 11A | 22256640 | $1^2 11^2$ |
| 12A | 20401920 | $2^1 4^1 6^1 12^1$ |
| 12B | 20401920 | 12^2 |
| 14A | 17487360 | $1^1 2^1 7^1 14^1$ |
| 14B | 17487360 | $1^1 2^1 7^1 14^1$ |
| 15A | 16321536 | $1^1 3^1 5^1 15^1$ |
| 15B | 16321536 | $1^1 3^1 5^1 15^1$ |
| 21A | 11658240 | $3^1 21^1$ |
| 21B | 11658240 | $3^1 21^1$ |
| 23A | 10644480 | $1^1 23^1$ |
| 23B | 10644480 | $1^1 23^1$ |

TABLE 2. Cycle shapes of conjugacy classes of M_{24} .

TABLE 3. multiplicities of the decomposition of $A(n)$ into irreducible representations of M_{24} in Mathieu moonshine

| n | χ_1 | χ_2 | $\chi_3 = \chi_4$ | $\chi_5 = \chi_6$ | χ_7 | χ_8 | χ_9 | $\chi_{10} = \chi_{11}$ | $\chi_{12} = \chi_{13}$ | χ_{14} | $\chi_{15} = \chi_{16}$ | χ_{17} | χ_{18} | χ_{19} | χ_{20} | χ_{21} | χ_{22} | χ_{23} | χ_{24} | χ_{25} | χ_{26} |
|-----|----------|----------|-------------------|-------------------|----------|----------|----------|-------------------------|-------------------------|-------------|-------------------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 0 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 2 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 2 | 2 | 2 | 2 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 2 | 0 | 0 | 2 | 2 | 2 | 4 | 2 | 2 | 6 |
| 9 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 0 | 2 | 2 | 0 | 2 | 2 | 2 | 4 | 4 | 4 | 8 | 8 | 10 |
| 10 | 0 | 0 | 0 | 2 | 0 | 2 | 2 | 0 | 2 | 2 | 2 | 4 | 4 | 4 | 6 | 6 | 8 | 12 | 10 | 10 | 24 |
| 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 4 | 6 | 4 | 2 | 8 | 10 | 8 | 14 | 12 | 22 | 24 | 26 | 40 |
| 12 | 0 | 2 | 0 | 2 | 2 | 4 | 4 | 6 | 8 | 4 | 8 | 12 | 12 | 12 | 18 | 26 | 30 | 40 | 38 | 40 | 80 |
| 13 | 0 | 0 | 2 | 2 | 4 | 2 | 6 | 10 | 14 | 18 | 14 | 16 | 26 | 30 | 28 | 44 | 44 | 70 | 80 | 84 | 136 |
| 14 | 0 | 0 | 0 | 8 | 4 | 6 | 14 | 16 | 24 | 22 | 24 | 34 | 38 | 46 | 58 | 80 | 86 | 128 | 126 | 132 | 254 |
| 15 | 0 | 0 | 2 | 8 | 12 | 8 | 18 | 38 | 40 | 46 | 44 | 46 | 78 | 86 | 88 | 138 | 144 | 218 | 238 | 246 | 424 |
| 16 | 0 | 2 | 2 | 18 | 18 | 22 | 36 | 50 | 72 | 68 | 72 | 100 | 122 | 140 | 170 | 232 | 252 | 378 | 382 | 400 | 742 |
| 17 | 0 | 2 | 8 | 25 | 30 | 26 | 54 | 94 | 116 | 130 | 124 | 140 | 212 | 246 | 262 | 392 | 410 | 630 | 670 | 704 | 1222 |
| 18 | 0 | 6 | 6 | 50 | 50 | 58 | 100 | 148 | 194 | 192 | 202 | 256 | 342 | 388 | 454 | 654 | 704 | 1044 | 1074 | 1120 | 2058 |
| 19 | 0 | 4 | 18 | 68 | 80 | 72 | 150 | 252 | 318 | 346 | 332 | 394 | 582 | 664 | 722 | 1062 | 1116 | 1702 | 1800 | 1880 | 3320 |
| 20 | 0 | 14 | 20 | 126 | 128 | 138 | 254 | 390 | 516 | 520 | 536 | 676 | 904 | 1036 | 1196 | 1716 | 1836 | 2764 | 2846 | 2980 | 5408 |
| 21 | 2 | 20 | 40 | 182 | 214 | 200 | 396 | 652 | 814 | 872 | 860 | 1020 | 1476 | 1684 | 1862 | 2742 | 2902 | 4384 | 4622 | 4828 | 8572 |
| 22 | 2 | 32 | 55 | 314 | 328 | 346 | 640 | 988 | 1298 | 1336 | 1348 | 1686 | 2302 | 2630 | 3000 | 4324 | 4616 | 6950 | 7204 | 7532 | 13620 |
| 23 | 2 | 40 | 98 | 460 | 512 | 496 | 972 | 1590 | 2020 | 2144 | 2118 | 2546 | 3638 | 4162 | 4624 | 6768 | 7166 | 10856 | 11376 | 11898 | 21204 |
| 24 | 0 | 80 | 132 | 744 | 798 | 824 | 1544 | 2426 | 3140 | 3236 | 3278 | 4050 | 5584 | 6376 | 7248 | 10500 | 11192 | 16834 | 17504 | 18294 | 32976 |
| 25 | 8 | 108 | 234 | 1106 | 1232 | 1208 | 2336 | 3764 | 4814 | 5084 | 5038 | 6108 | 8654 | 9892 | 11042 | 16112 | 17084 | 25840 | 27056 | 28288 | 50524 |
| 26 | 6 | 174 | 322 | 1742 | 1860 | 1904 | 3602 | 5677 | 7348 | 7626 | 7670 | 9444 | 13090 | 14968 | 16940 | 24566 | 26148 | 39428 | 41022 | 42894 | 77176 |
| 27 | 12 | 252 | 514 | 2560 | 2836 | 2802 | 5394 | 8688 | 11092 | 11666 | 11618 | 14100 | 19914 | 22744 | 25462 | 37148 | 39436 | 59564 | 62294 | 65114 | 116494 |
| 28 | 16 | 398 | 742 | 3922 | 4238 | 4310 | 8160 | 12912 | 16686 | 17356 | 17418 | 21414 | 29772 | 34026 | 38434 | 55764 | 59330 | 89490 | 93218 | 97456 | 175146 |
| 29 | 26 | 560 | 1154 | 5758 | 6328 | 6286 | 12090 | 19380 | 24840 | 26078 | 25994 | 31636 | 44512 | 50892 | 57068 | 83146 | 88280 | 133356 | 139342 | 145690 | 260828 |
| 30 | 34 | 876 | 1642 | 8642 | 9368 | 9486 | 18008 | 28580 | 36824 | 38368 | 38480 | 47172 | 65776 | 75158 | 84776 | 123176 | 131020 | 197596 | 205970 | 215318 | 386724 |

TABLE 4. Expansion coefficients of $A_g(n)$ in Mathieu moonshine.

| n | 1A | 2A | 2B | 3A | 3B | 4A | 4B | 4C | 5A | 6A | 6B | 7AB | 8A | 10A | 11A | 12A | 12B | 14AB | 15AB | 21AB | 23AB |
|-----|------------|--------|--------|------|------|-----|-----|-----|-----|-----|-----|-----|----|-----|-----|-----|-----|------|------|------|------|
| 0 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 | -2 |
| 1 | 90 | -6 | 10 | 0 | 6 | -6 | 2 | 2 | 0 | 0 | -2 | -1 | -2 | 0 | 2 | 0 | 2 | 1 | 0 | -1 | -2 |
| 2 | 462 | 14 | -18 | -6 | 0 | -2 | -2 | 6 | 2 | 2 | 0 | 0 | -2 | 2 | 0 | -2 | 0 | 0 | -1 | 0 | 2 |
| 3 | 1540 | -28 | 20 | 10 | -14 | 4 | -4 | -4 | 0 | 2 | 2 | 0 | 0 | 0 | 0 | -2 | 2 | 0 | 0 | 0 | -1 |
| 4 | 4554 | 42 | -38 | 0 | 12 | -6 | 2 | -6 | -6 | 0 | 4 | 4 | -2 | 2 | 0 | 0 | 0 | 0 | 0 | -2 | 0 |
| 5 | 11592 | -56 | 72 | -18 | 0 | -8 | 8 | 0 | 2 | -2 | 0 | 0 | 0 | 2 | -2 | -2 | 0 | 0 | 2 | 0 | 0 |
| 6 | 27830 | 86 | -90 | 20 | -16 | 6 | -2 | 6 | 0 | -4 | 0 | -2 | 2 | 0 | 0 | 0 | 0 | 2 | 0 | -2 | 0 |
| 7 | 61686 | -138 | 118 | 0 | 30 | 6 | -10 | -2 | 6 | 0 | -2 | 2 | -2 | -2 | -2 | 0 | -2 | 2 | 0 | 2 | 0 |
| 8 | 131100 | 188 | -180 | -30 | 0 | -4 | 4 | -12 | 0 | 2 | 0 | -3 | 0 | 0 | 2 | 2 | 0 | -1 | 0 | 0 | 0 |
| 9 | 265650 | -238 | 258 | 42 | -42 | -14 | 10 | 10 | -10 | 2 | 6 | 0 | -2 | -2 | 0 | -2 | -2 | 0 | 2 | 0 | 0 |
| 10 | 521136 | 336 | -352 | 0 | 42 | 0 | -8 | 16 | 6 | 0 | 2 | 0 | -4 | -2 | 0 | 0 | -2 | 0 | 0 | 0 | 2 |
| 11 | 988770 | -478 | 450 | -60 | 0 | 18 | -14 | -6 | 0 | -4 | 0 | 6 | 2 | 0 | 2 | 0 | 0 | -2 | 0 | 0 | 0 |
| 12 | 1830248 | 616 | -600 | 62 | -70 | -8 | 8 | -16 | 8 | -2 | -6 | 0 | 0 | 0 | 2 | -2 | 2 | 0 | 2 | 0 | 0 |
| 13 | 3303630 | -786 | 830 | 0 | 84 | -18 | 22 | 6 | 0 | 0 | -4 | -6 | 2 | 0 | 0 | 0 | 0 | -2 | 0 | 0 | 2 |
| 14 | 5844762 | 1050 | -1062 | -90 | 0 | 10 | -6 | 18 | -18 | 6 | 0 | 0 | 2 | -2 | 0 | -2 | 0 | 0 | 0 | 0 | 2 |
| 15 | 10139734 | -1386 | 1334 | 118 | -110 | 22 | -26 | -10 | 4 | 6 | 2 | -4 | -2 | 4 | 0 | -2 | 2 | 0 | -2 | 2 | 0 |
| 16 | 17301060 | 1764 | -1740 | 0 | 126 | -12 | 12 | -28 | 0 | 0 | 6 | 0 | 0 | 0 | -4 | 0 | 2 | 0 | 0 | 0 | 0 |
| 17 | 29051484 | -2212 | 2268 | -156 | 0 | -36 | 28 | 12 | 14 | -4 | 0 | 0 | -4 | -2 | 0 | 0 | 0 | 0 | -1 | 0 | 0 |
| 18 | 48106430 | 2814 | -2850 | 170 | -166 | 14 | -18 | 38 | 0 | -6 | -6 | 8 | -2 | 0 | -2 | 2 | 2 | 0 | 0 | 2 | -2 |
| 19 | 78599556 | -3612 | 3540 | 0 | 210 | 36 | -36 | -20 | -24 | 0 | -6 | 0 | 0 | 0 | 2 | 0 | -2 | 0 | 0 | 0 | 0 |
| 20 | 126894174 | 4510 | -4482 | -228 | 0 | -18 | 14 | -42 | 14 | 4 | 0 | -6 | -2 | -2 | 0 | 0 | 0 | 2 | 2 | 0 | 0 |
| 21 | 202537080 | -5544 | 5640 | 270 | -282 | -40 | 48 | 16 | 0 | 6 | 6 | 4 | 4 | 0 | -2 | 2 | -2 | 0 | 0 | -2 | 0 |
| 22 | 319927608 | 6936 | -6968 | 0 | 300 | 24 | -16 | 48 | 18 | 0 | 4 | -7 | 4 | 2 | 0 | 0 | 0 | -1 | 0 | -1 | 0 |
| 23 | 500376870 | -8666 | 8550 | -360 | 0 | 54 | -58 | -18 | 0 | -8 | 0 | 0 | -2 | 0 | 4 | 0 | 0 | 0 | 0 | 0 | 2 |
| 24 | 775492564 | 10612 | -10556 | 400 | -392 | -28 | 28 | -60 | -36 | -8 | -8 | 0 | 0 | 4 | 0 | -4 | 0 | 0 | 0 | 0 | 0 |
| 25 | 1191453912 | -12936 | 13064 | 0 | 462 | -72 | 64 | 32 | 12 | 0 | -10 | 12 | -4 | 4 | 0 | 0 | 2 | 0 | 0 | 0 | 0 |
| 26 | 1815754710 | 15862 | -15930 | -510 | 0 | 22 | -34 | 78 | 0 | 10 | 0 | 0 | -6 | 0 | 0 | -2 | 0 | 0 | 0 | 0 | -1 |
| 27 | 2745870180 | -19420 | 19268 | 600 | -600 | 84 | -76 | -36 | 30 | 8 | 8 | -10 | 4 | -2 | -2 | 0 | 0 | -2 | 0 | 2 | 0 |
| 28 | 4122417420 | 23532 | -23460 | 0 | 660 | -36 | 36 | -84 | 0 | 0 | 12 | 2 | 0 | 0 | 0 | 0 | 0 | -2 | 0 | 2 | 0 |
| 29 | 6146311620 | -28348 | 28548 | -762 | 0 | -92 | 100 | 36 | -50 | -10 | 0 | -6 | 4 | -2 | -2 | -2 | 0 | 2 | -2 | 0 | 0 |
| 30 | 9104078592 | 34272 | -34352 | 828 | -840 | 48 | -40 | 96 | 22 | -12 | -8 | 0 | 4 | -2 | 4 | 0 | 0 | 0 | -2 | 0 | 0 |

| g | size | cycle shape |
|-----|-------|-------------------|
| 1A | 1 | 1^{12} |
| 2A | 396 | 2^6 |
| 2B | 495 | $1^4 2^4$ |
| 3A | 1760 | $1^3 3^3$ |
| 3B | 2640 | 3^4 |
| 4A | 2970 | $2^2 4^2$ |
| 4B | 2970 | $1^4 4^2$ |
| 5A | 9504 | $1^2 5^2$ |
| 6A | 7920 | 6^2 |
| 6B | 15840 | $1^1 2^1 3^1 6^1$ |
| 8A | 11880 | $4^1 8^1$ |
| 8B | 11880 | $1^2 2^1 8^1$ |
| 10A | 9504 | $2^1 10^1$ |
| 11A | 8640 | $1^1 11^1$ |
| 11B | 8640 | $1^1 11^1$ |

TABLE 5. Cycle shapes of conjugacy classes of M_{12} .

| | 1A | 2A | 2B | 3A | 3B | 4A | 4B | 5A | 6A | 6B | 8A | 8B | 10A | 11A | 11B |
|-------------|-----|----|----|----|----|----|----|----|----|----|----|----|-----|---------------------------|---------------------------|
| χ_1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| χ_2 | 11 | -1 | 3 | 2 | -1 | -1 | 3 | 1 | -1 | 0 | -1 | 1 | -1 | 0 | 0 |
| χ_3 | 11 | -1 | 3 | 2 | -1 | 3 | -1 | 1 | -1 | 0 | 1 | -1 | -1 | 0 | 0 |
| χ_4 | 16 | 4 | 0 | -2 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | -1 | $\frac{-1+i\sqrt{11}}{2}$ | $\frac{-1-i\sqrt{11}}{2}$ |
| χ_5 | 16 | 4 | 0 | -2 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | -1 | $\frac{-1-i\sqrt{11}}{2}$ | $\frac{-1+i\sqrt{11}}{2}$ |
| χ_6 | 45 | 5 | -3 | 0 | 3 | 1 | 1 | 0 | -1 | 0 | -1 | -1 | 0 | 1 | 1 |
| χ_7 | 54 | 6 | 6 | 0 | 0 | 2 | 2 | -1 | 0 | 0 | 0 | 0 | 1 | -1 | -1 |
| χ_8 | 55 | -5 | 7 | 1 | 1 | -1 | -1 | 0 | 1 | 1 | -1 | -1 | 0 | 0 | 0 |
| χ_9 | 55 | -5 | -1 | 1 | 1 | 3 | -1 | 0 | 1 | -1 | -1 | 1 | 0 | 0 | 0 |
| χ_{10} | 55 | -5 | -1 | 1 | 1 | -1 | 3 | 0 | 1 | -1 | 1 | -1 | 0 | 0 | 0 |
| χ_{11} | 66 | 6 | 2 | 3 | 0 | -2 | -2 | 1 | 0 | -1 | 0 | 0 | 1 | 0 | 0 |
| χ_{12} | 99 | -1 | 3 | 0 | 3 | -1 | -1 | -1 | -1 | 0 | 1 | 1 | -1 | 0 | 0 |
| χ_{13} | 120 | 0 | -8 | 3 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | -1 | -1 |
| χ_{14} | 144 | 4 | 0 | 0 | -3 | 0 | 0 | -1 | 1 | 0 | 0 | 0 | -1 | 1 | 1 |
| χ_{15} | 176 | -4 | 0 | -4 | -1 | 0 | 0 | 1 | -1 | 0 | 0 | 0 | 1 | 0 | 0 |

TABLE 6. character table of M_{12} . $|M_{12}| = 95040$

| $M_{24} \setminus M_{12}$ | χ_1 | χ_2 | χ_3 | χ_4 | χ_5 | χ_6 | χ_7 | χ_8 | χ_9 | χ_{10} | χ_{11} | χ_{12} | χ_{13} | χ_{14} | χ_{15} |
|---------------------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|-------------|-------------|-------------|-------------|-------------|-------------|
| | 1 | 11 | 11 | 16 | 16 | 45 | 54 | 55 | 55 | 55 | 66 | 99 | 120 | 144 | 176 |
| χ_1 | 1 | 1 | | | | | | | | | | | | | |
| χ_2 | 23 | 1 | 1 | 1 | | | | | | | | | | | |
| χ_3 | 45 | | | | | 1 | | | | | | | | | |
| χ_4 | 45 | | | | | 1 | | | | | | | | | |
| χ_5 | 231 | | | | | | | 1 | | | | | | | 1 |
| χ_6 | 231 | | | | | | | 1 | | | | | | | 1 |
| χ_7 | 252 | 1 | 1 | 1 | | | 2 | 1 | | | 1 | | | | |
| χ_8 | 253 | | 1 | 1 | | | | 1 | 1 | 1 | 1 | | | | |
| χ_9 | 483 | | 1 | 1 | | | 2 | 2 | | | | 1 | | 1 | |
| χ_{10} | 770 | | | | | | | | | | 1 | | 2 | 2 | 1 |
| χ_{11} | 770 | | | | | | | | | | 1 | | 2 | 2 | 1 |
| χ_{12} | 990 | | | | | 1 | | | 1 | 1 | | 1 | 2 | 1 | 2 |
| χ_{13} | 990 | | | | | 1 | | | 1 | 1 | | 1 | 2 | 1 | 2 |
| χ_{14} | 1035 | 1 | | | 1 | 1 | 2 | 1 | | | 2 | 2 | | 2 | 1 |
| χ_{15} | 1035 | | | | | 1 | | | 1 | 1 | | | 2 | 2 | 2 |
| χ_{16} | 1035 | | | | | 1 | | | 1 | 1 | | | 2 | 2 | 2 |
| χ_{17} | 1265 | 1 | 1 | 1 | | | 2 | 3 | 1 | 1 | 1 | 3 | | 1 | 2 |
| χ_{18} | 1771 | | | | | 2 | | 1 | 1 | 1 | 3 | 2 | 4 | 2 | 2 |
| χ_{19} | 2024 | | | 1 | 1 | 2 | 2 | 1 | 1 | 1 | 2 | 3 | 2 | 3 | 3 |
| χ_{20} | 2277 | | | | | 1 | 2 | 3 | 2 | 2 | 1 | 3 | 2 | 3 | 4 |
| χ_{21} | 3312 | | 1 | 1 | | 1 | 4 | 3 | 1 | 1 | 3 | 4 | 2 | 6 | 6 |
| χ_{22} | 3520 | | 2 | 2 | | | 4 | 4 | 2 | 2 | 4 | 4 | 2 | 6 | 6 |
| χ_{23} | 5313 | | 1 | 1 | 2 | 2 | 2 | 4 | 5 | 2 | 2 | 4 | 6 | 4 | 8 |
| χ_{24} | 5544 | | | 1 | 1 | 4 | 2 | 1 | 3 | 3 | 4 | 5 | 10 | 9 | 9 |
| χ_{25} | 5796 | | | 2 | 2 | 4 | 4 | 1 | 3 | 3 | 4 | 5 | 8 | 9 | 11 |
| χ_{26} | 10395 | | 1 | 1 | 1 | 1 | 4 | 4 | 6 | 7 | 7 | 6 | 11 | 14 | 15 |

TABLE 7. Branching of M_{24} representations into those of M_{12} . Only non-zero multiplicities are written.

| n | χ_1 | $\chi_2 = \chi_3$ | $\chi_4 = \chi_5$ | χ_6 | χ_7 | χ_8 | $\chi_9 = \chi_{10}$ | χ_{11} | χ_{12} | χ_{13} | χ_{14} | χ_{15} |
|-----|----------|-------------------|-------------------|----------|----------|----------|----------------------|-------------|-------------|-------------|-------------|-------------|
| 0 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 2 | 2 | 1 |
| 4 | 0 | 0 | 0 | 1 | 2 | 3 | 2 | 1 | 3 | 2 | 3 | 4 |
| 5 | 0 | 0 | 2 | 4 | 4 | 1 | 3 | 4 | 5 | 8 | 9 | 11 |
| 6 | 0 | 3 | 1 | 4 | 8 | 10 | 9 | 10 | 15 | 16 | 21 | 26 |
| 7 | 0 | 2 | 7 | 18 | 16 | 15 | 17 | 23 | 32 | 42 | 46 | 56 |
| 8 | 1 | 9 | 10 | 28 | 38 | 43 | 39 | 43 | 70 | 78 | 98 | 124 |
| 9 | 1 | 14 | 23 | 66 | 76 | 70 | 75 | 94 | 134 | 174 | 206 | 242 |
| 10 | 3 | 33 | 42 | 119 | 148 | 162 | 154 | 179 | 276 | 322 | 390 | 485 |
| 11 | 4 | 51 | 88 | 242 | 278 | 272 | 282 | 346 | 511 | 632 | 753 | 914 |
| 12 | 10 | 115 | 147 | 420 | 522 | 546 | 534 | 633 | 956 | 1144 | 1384 | 1699 |
| 13 | 19 | 183 | 286 | 801 | 938 | 933 | 951 | 1152 | 1716 | 2102 | 2506 | 3051 |
| 14 | 30 | 346 | 484 | 1364 | 1664 | 1721 | 1698 | 2018 | 3056 | 3666 | 4420 | 5423 |
| 15 | 52 | 576 | 861 | 2420 | 2874 | 2896 | 2922 | 3535 | 5263 | 6434 | 7697 | 9375 |
| 16 | 94 | 1017 | 1444 | 4069 | 4922 | 5058 | 5022 | 5994 | 9033 | 10886 | 13087 | 16032 |
| 17 | 151 | 1658 | 2468 | 6920 | 8248 | 8340 | 8388 | 10099 | 15107 | 18382 | 22027 | 26887 |
| 18 | 252 | 2817 | 4020 | 11330 | 13674 | 14000 | 13941 | 16689 | 25077 | 30316 | 36427 | 44563 |
| 19 | 412 | 4508 | 6647 | 18681 | 22316 | 22644 | 22717 | 27318 | 40913 | 49696 | 59567 | 72744 |
| 20 | 669 | 7385 | 10649 | 29960 | 36064 | 36844 | 36750 | 44021 | 66134 | 80010 | 96094 | 117541 |
| 21 | 1064 | 11676 | 17087 | 48040 | 57526 | 58442 | 58560 | 70371 | 105420 | 127988 | 153496 | 187481 |
| 22 | 1692 | 18579 | 26877 | 75625 | 90908 | 92775 | 92630 | 111037 | 166710 | 201830 | 242298 | 296284 |
| 23 | 2622 | 28863 | 42197 | 118616 | 142120 | 144536 | 144714 | 173798 | 260529 | 316064 | 379145 | 463254 |
| 24 | 4082 | 44995 | 65174 | 183384 | 220348 | 224690 | 224472 | 269200 | 403992 | 489368 | 587424 | 718126 |
| 25 | 6270 | 68818 | 100406 | 282327 | 338446 | 344382 | 344655 | 413792 | 620437 | 752450 | 902705 | 1103084 |
| 26 | 9555 | 105225 | 152718 | 429576 | 515886 | 525845 | 525510 | 630341 | 945863 | 1145966 | 1375439 | 1681406 |
| 27 | 14433 | 158731 | 231277 | 650388 | 780008 | 793968 | 794367 | 953589 | 1429925 | 1733926 | 2080389 | 2542299 |
| 28 | 21711 | 238790 | 346819 | 975551 | 1171218 | 1193511 | 1193023 | 1431222 | 2147351 | 2602046 | 3122821 | 3817239 |
| 29 | 32314 | 355395 | 517616 | 1455614 | 1746034 | 1777621 | 1778220 | 2134316 | 3200923 | 3880816 | 4656537 | 5690817 |
| 30 | 47909 | 527223 | 766024 | 2154660 | 2586488 | 2635260 | 2634546 | 3160915 | 4742013 | 5746832 | 6896777 | 8429971 |

TABLE 8. multiplicities of irreducible representations of M_{12} in Enriques moonshine

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